Prove the derivative of $\cos^{-1} x$. **Show all steps and justifications.** SCORE: _____ / 4 PTS

$y = \cos^{-1} x \longrightarrow y \in [0, \pi] \longrightarrow \sin y \geq 0$ 1

$\cos y = x$ 1

so $\sin y = \sqrt{1 - \cos^2 y}$ $\frac{1}{2}$

$= \sqrt{1 - x^2}$

$-\sin y \frac{dy}{dx} = 1$ $\frac{1}{2}$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1 - x^2}}$ $\frac{1}{2}$ $\frac{1}{2}$

Find $\dfrac{dy}{dx}$ if $\cos\dfrac{y}{x} = x^2y^3$.

**SCORE: _____ / 6 PTS**

$$\boxed{\left(-\sin\frac{y}{x}\right)}\boxed{\left(\frac{y'x-y}{x^2}\right)} = \boxed{2xy^3} + \boxed{3x^2y^2y'}$$

1 1 1 1

$$\left(-\sin\frac{y}{x}\right)(y'x-y) = 2x^3y^3 + 3x^4y^2y'$$

$$\left(-x\sin\frac{y}{x}\right)y' + y\sin\frac{y}{x} = 2x^3y^3 + 3x^4y^2y'$$

$$\boxed{y\sin\frac{y}{x} - 2x^3y^3} = 3x^4y^2y' + \left(x\sin\frac{y}{x}\right)y'$$

1

$$\boxed{= \left(3x^4y^2 + x\sin\frac{y}{x}\right)y'}$$

$$\frac{dy}{dx} = \boxed{\frac{y\sin\frac{y}{x} - 2x^3y^3}{3x^4y^2 + x\sin\frac{y}{x}}}$$

1

TALK TO ME IF YOU REWROTE THE EQUATION FIRST

The table shows values of $f(x)$ and $f'(x)$ at various input values. If $g(x) = \dfrac{f(x^2)}{x}$, find $g'(3)$. **SCORE: _____ / 4 PTS**

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | –4 | –1 | 0 | 2 | 1 | 4 | 6 | 5 | 3 | –3 |
| $f'(x)$ | 3 | 2 | 4 | 0 | 5 | 1 | –1 | –3 | –7 | –6 |

$$g'(x) = \boxed{\frac{f'(x^2)\cdot 2x\cdot x - f(x^2)}{x^2}} \quad 2\tfrac{1}{2}$$

THIS PRODUCT CAN BE WRITTEN DIFFERENTLY, AS LONG AS IT EQUALS –108

$$g'(3) = \frac{f'(9)\cdot 6\cdot 3 - f(9)}{9} = \boxed{\frac{\overbrace{(-6)18}^{2} - (-3)}{9}} = \boxed{-\frac{35}{3}}$$

$\tfrac{1}{2}$

IF YOUR FINAL ANSWER IS CORRECT, GIVE YOURSELF THIS POINT AUTOMATICALLY ⟶ 1

If $f(x)=\ln(x+\sqrt{x^2-1})$, find $f'(x)$. **NOTE: In the final simplified answer, $x$ appears only once.** SCORE: _____ / 5 PTS

$$f'(x)=\frac{1}{x+\sqrt{x^2-1}}\cdot\left(1+\frac{1}{\cancel{2}\sqrt{x^2-1}}\cdot\cancel{2}x\right)$$

$$=\boxed{\frac{1}{x+\sqrt{x^2-1}}\cdot\left(1+\frac{x}{\sqrt{x^2-1}}\right)}\ 2$$

1

$$=\frac{1}{x+\sqrt{x^2-1}}\cdot\boxed{\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}}\ 1$$

$$=\boxed{\frac{1}{\sqrt{x^2-1}}}\ 1$$

If $f(x) = (\sec x)^{x^3}$, find $f'(x)$. **SCORE: _____ / 6 PTS**

$y = (\sec x)^{x^3}$

$\ln y = x^3 \ln \sec x$ $1\frac{1}{2}$

$1$ $\frac{1}{y}\frac{dy}{dx} = 3x^2 \ln \sec x + x^3 \frac{1}{\cancel{\sec x}} \cancel{\sec x} \tan x$ $2\frac{1}{2}$

$\frac{dy}{dx} = (\sec x)^{x^3}(3x^2 \ln \sec x + x^3 \tan x)$ $1$

$= x^2 (\sec x)^{x^3}(3 \ln \sec x + x \tan x)$

If the position function of an object is given by $f(t) = \tan^{-1} t^4$, find the acceleration function. **SCORE: _____ / 5 PTS**

$$f'(t) = \frac{1}{1+(t^4)^2} 4t^3 = \boxed{\frac{4t^3}{1+t^8}}$$ 2

$$f''(t) = \boxed{\frac{12t^2(1+t^8) - 4t^3(8t^7)}{(1+t^8)^2}}$$ 2

$$= \frac{12t^2 + 12t^{10} - 32t^{10}}{(1+t^8)^2}$$

$$= \boxed{\frac{12t^2 - 20t^{10}}{(1+t^8)^2}}$$ 1 $$= \frac{4t^2(3-5t^8)}{(1+t^8)^2}$$